FM – Generation and Detection

Reference

- Chapter 5.3, Carlson, communication Systems.
- NBFM
- WBFM

Generation of FM signals

NBFM (Narrowband FM)

– NBFM can directly be implemented based on its general equation.

Carrier frequency

Modulating signal

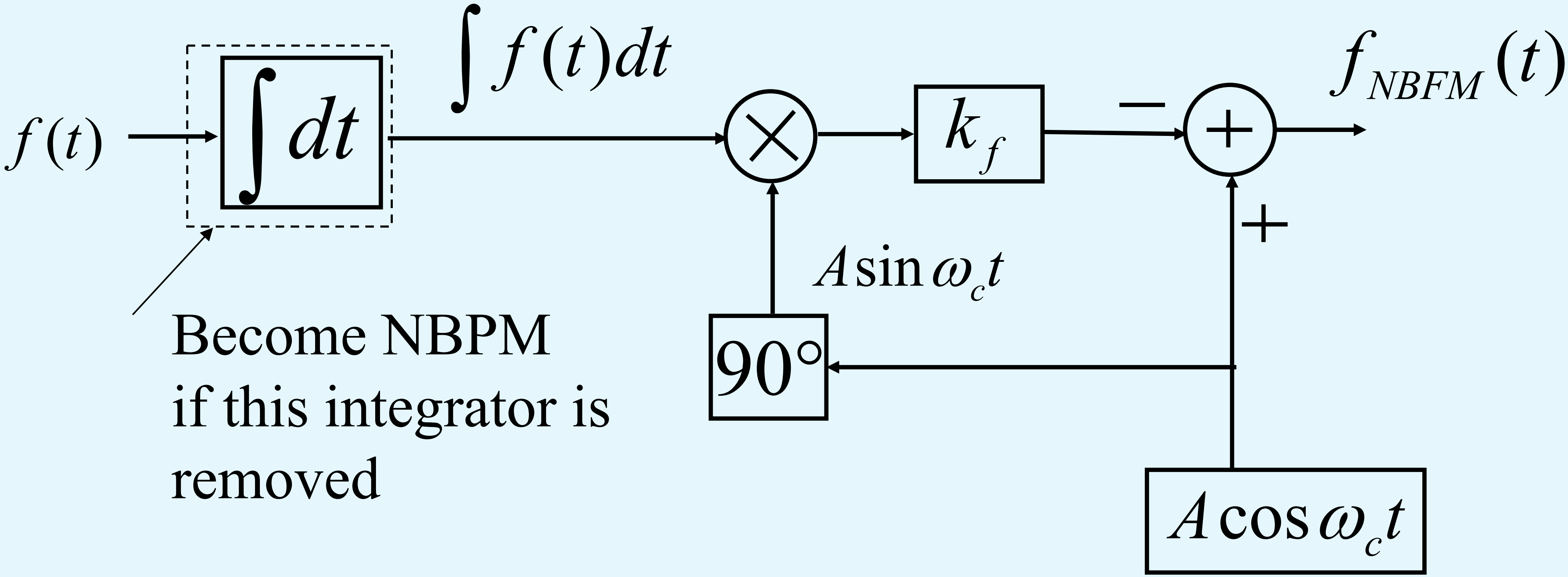
$$f_{NBFM}(t) = A\cos\left[\omega_c t + k_f\int_0^t f(\tau)d\tau\right]$$
$$= A\cos\omega_c t\cos\left[k_f\int_0^t f(\tau)d\tau\right] - A\sin\omega_c t\sin\left[k_f\int_0^t f(\tau)d\tau\right]$$
$$\approx A\cos\omega_c t - A\sin\omega_c t\cdot\left[k_f\int_0^t f(\tau)d\tau\right] \qquad \because \left|k_f\int_0^t f(\tau)d\tau\right| << 1$$

$\cos x \approx 1 \quad \text{if } x << 1$

$\sin x \approx x \quad \text{if } x << 1$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

NBFM

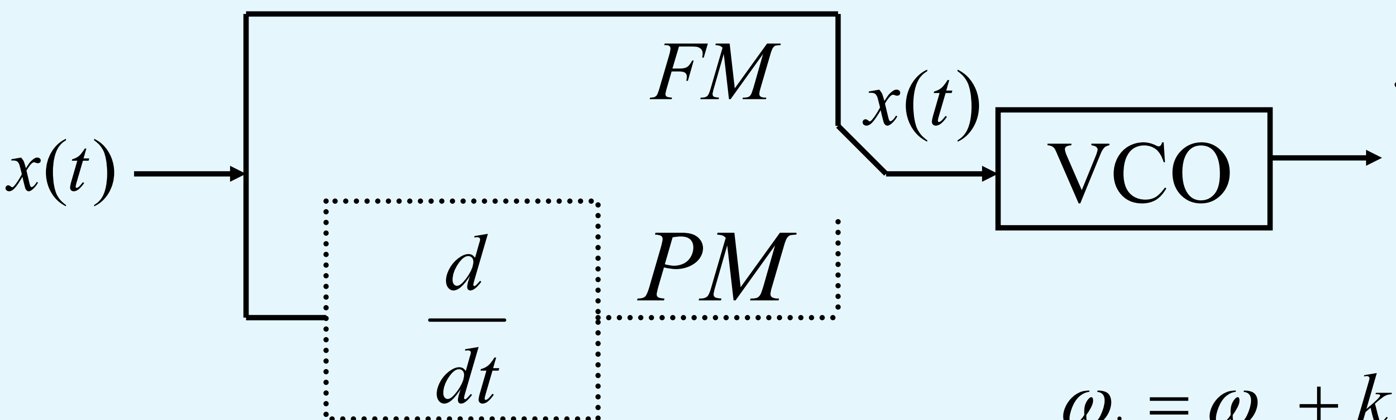


$$f_{PM} = A\cos\left[\omega_c t + k_p f(t)\right]$$

WBFM

A. Direct Method

- FM: $f_i \propto x(t)$
- Voltage-controlled oscillator (VCO)
  - $f_{out} \propto v_{in}$



$f_{FM}(t)$

$\omega_i = \omega_c + k_f x(t)$

$$x(t) = \frac{df(t)}{dt}$$

$$\omega_i = \omega_c + k_f \frac{df(t)}{dt}$$

$$\Rightarrow \theta_i = \int \omega_i dt = \omega_c t + k_f f(t) + \theta_o$$

WBFM

B. Indirect Method

– consists of a NBFM modulator, frequency multipliers and frequency converters.

– frequency multiplier

- nonlinear device
- multiply the frequencies of the input signal by a given factor
- Example: an ideal square-law device

$$e_o(t) = a e_i^2(t)$$

WBFM

$$e_o(t) = a e_i^2(t)$$

If the input signal is a FM signal,

$$e_i(t) = A\cos(\omega_c t + \beta \sin \omega_m t)$$

the output signal is

$$e_o(t) = aA^2 \cos^2(\omega_c t + \beta \sin \omega_m t)$$

$$= \frac{1}{2} aA^2 \left[1 + \cos(2\omega_c t + 2\beta \sin \omega_m t)\right]$$

Can be removed with a filter

A FM signal with carrier frequency $2\omega_c$ and modulation index 2β

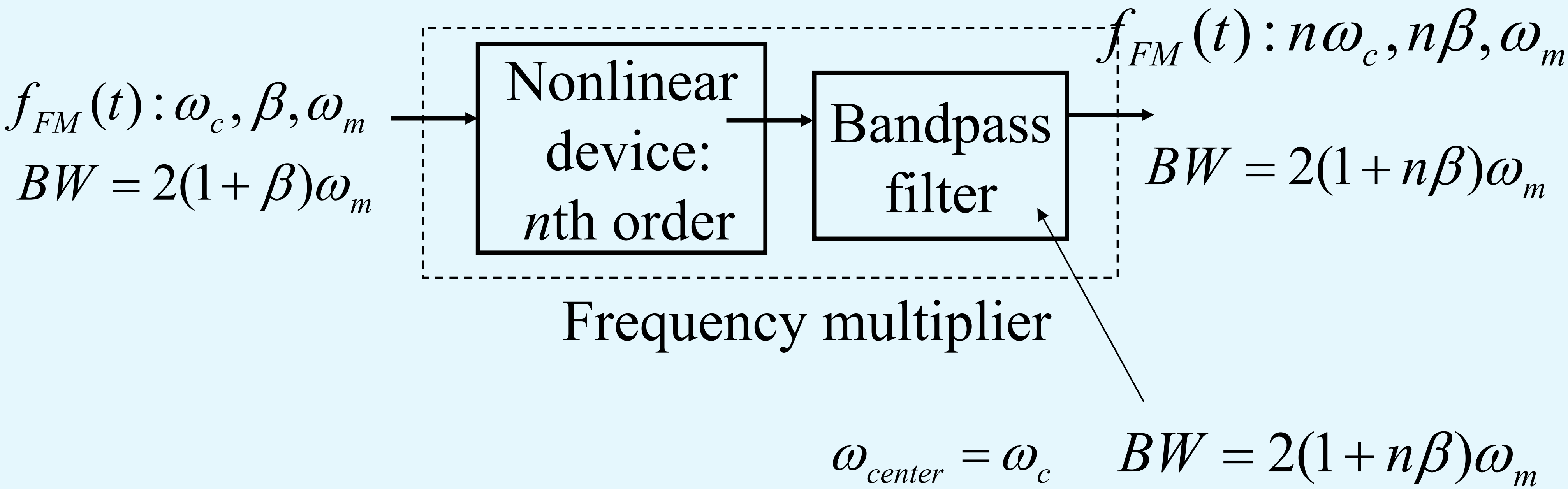
Modulating frequency is unchanged

WBFM

n^{th} law device followed by a filter

– new FM signal

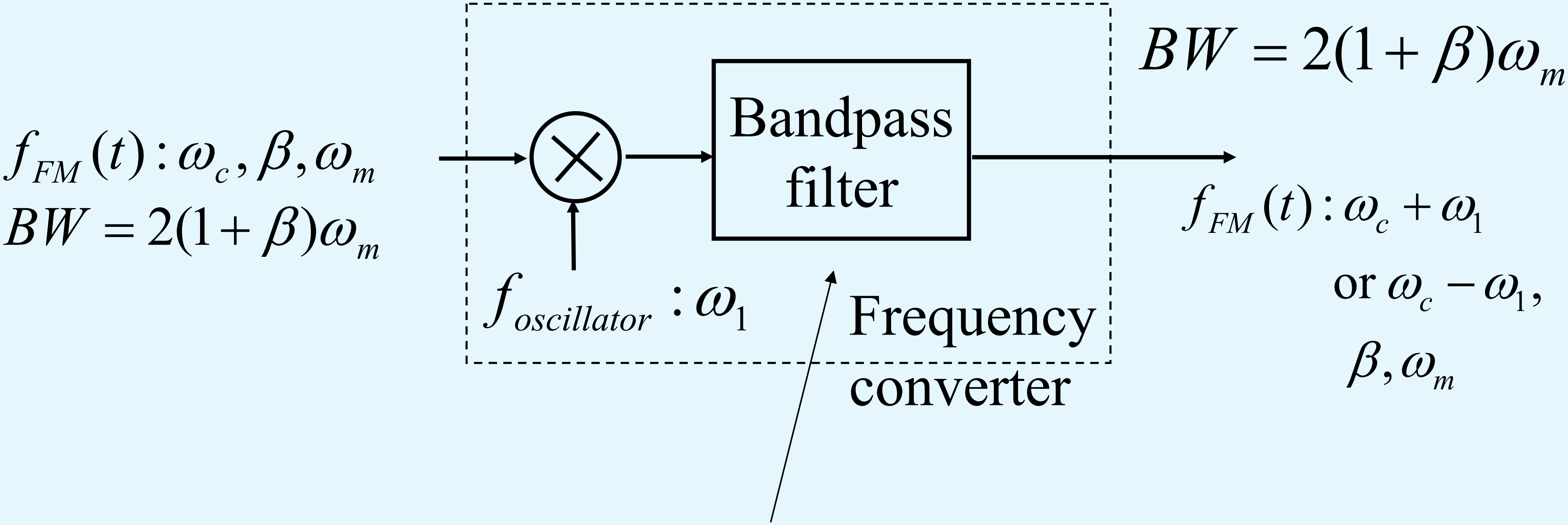
- carrier and a modulation index that have been increased by a factor of n.



WBFM

Frequency converter

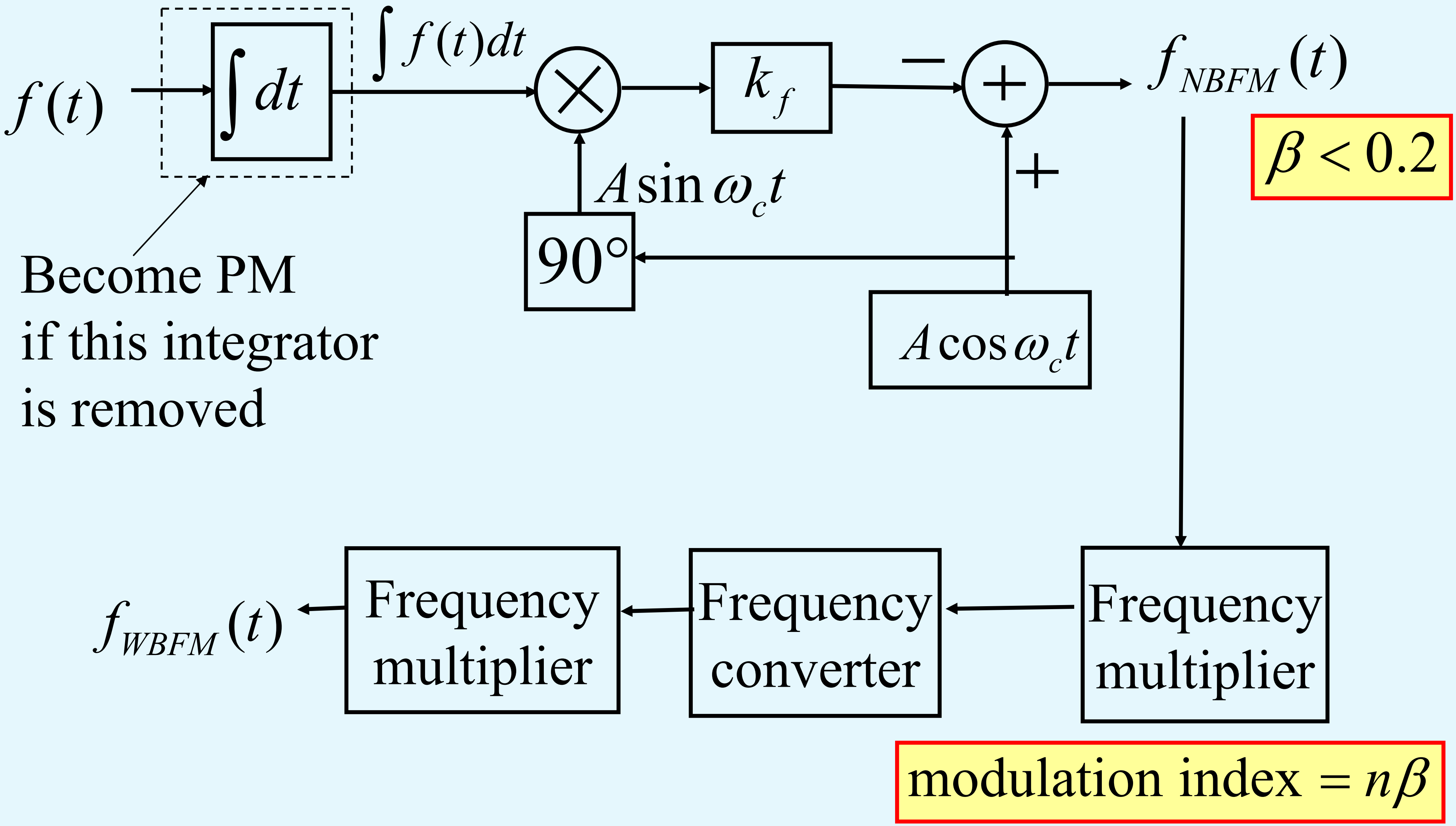
– to control the value of the carrier frequency.



$$\omega_{center} = \omega_c + \omega_1 \text{ or } \omega_c - \omega_1$$

$$BW = 2(1+\beta)\omega_m$$

WBFM

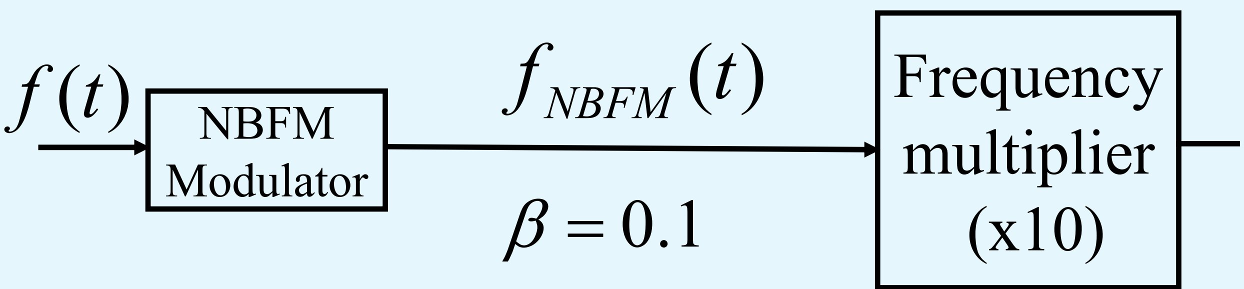


$f(t)$
$\int dt$
$\int f(t)dt$
k_f
$-$
$+$
$f_{NBFM}(t)$
$A\sin\omega_c t$
$90°$
$A\cos\omega_c t$
Become PM
if this integrator
is removed
$\beta < 0.2$
Frequency multiplier
Frequency converter
Frequency multiplier
$f_{WBFM}(t)$
modulation index $= n\beta$

WBFM

Example:

generate a FM signal with $f_c = 50\text{MHz}$ and $\beta = 1$ from a NBFM signal with $f_c = 1\text{MHz}$, $\beta = 0.1$ and $f_m = 10\text{kHz}$



$\beta = 0.1$

$f_m = 10kHz$

$f_c = 1MHz$

$\beta = 10 \times 0.1 = 1$

$f_c = 10 \times 1MHz = 10MHz$

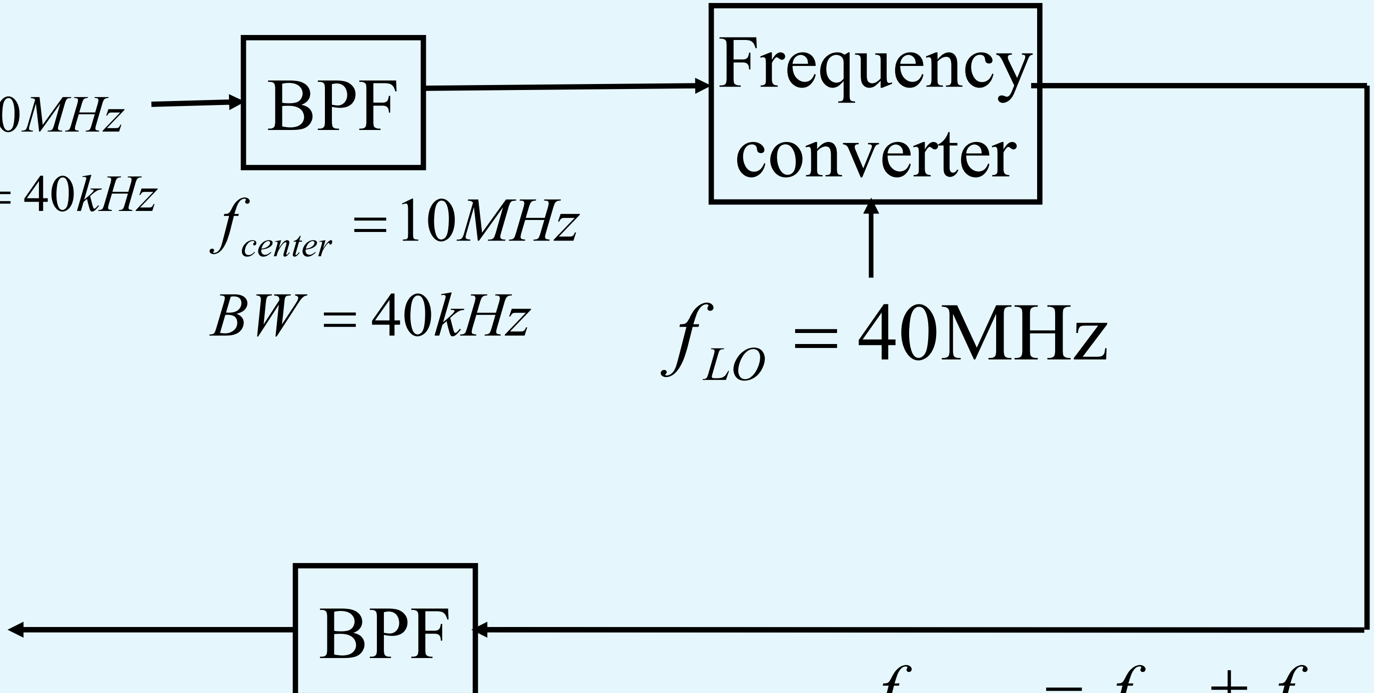
$\text{BW} = 2(1+\beta) f_m = 40kHz$

WBFM

$\beta = 10 \times 0.1 = 1$

$f_c = 10 \times 1MHz = 10MHz$

$\text{BW} = 2(1+\beta) f_m = 40kHz$



$f_{center} = 10MHz$

$BW = 40kHz$

$f_{LO} = 40\text{MHz}$

$f_{WBFM}(t)$

$f_{center} = 50MHz$

$\text{BW} = 40kHz$

$$f_{c(new)} = f_{LO} \pm f_c$$
$$= 40MHz \pm 10MHz$$
$$= 50MHz \text{ and } 30MHz$$
$$\text{BW} = 40kHz$$

Demodulation of FM signals

Direct Method

– FM signal:

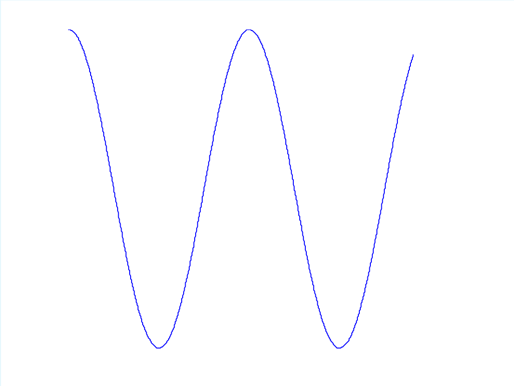
$$f_{FM}(t) = A\cos\left[\omega_c t + k_f \int_0^t f(\tau)d\tau\right]$$

$$\frac{d}{dt} f_{FM}(t) = -A\left[\omega_c + k_f f(t)\right]\cos\left[\omega_c t + k_f \int_0^t f(\tau)d\tau\right]$$

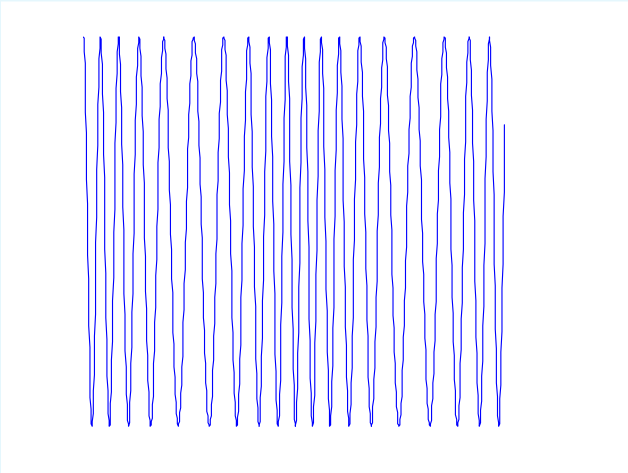
– If $k_f f(t) << \omega_c$, the equation is in the form of an AM signal whose envelope is

$$\approx A\omega_c\left[1 + \frac{k_f}{\omega_c} f(t)\right]$$

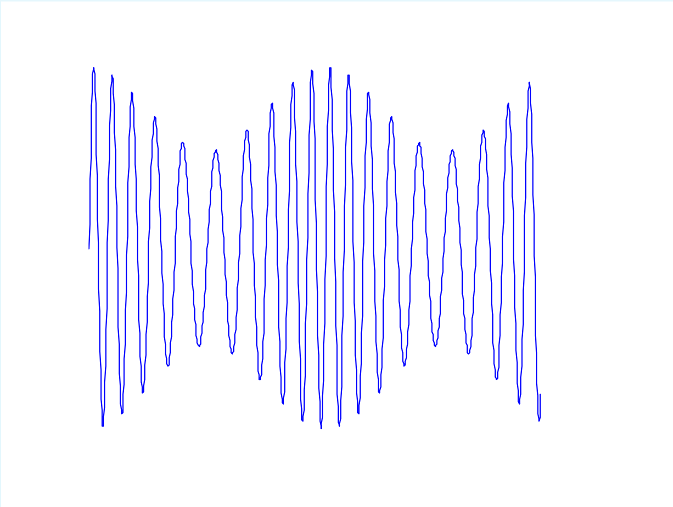
Demodulation of FM signals



$\cos \omega_m t$



$\cos\left[10\omega_m t + 3\sin \omega_m t\right]$

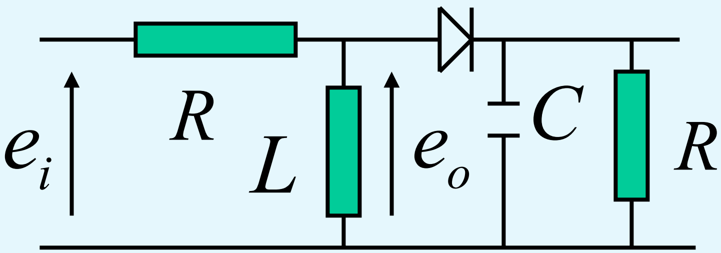


$\left[10\omega_m + 3\omega_m \cos \omega_m t\right]\sin\left[10\omega_m t + 3\sin \omega_m t\right]$

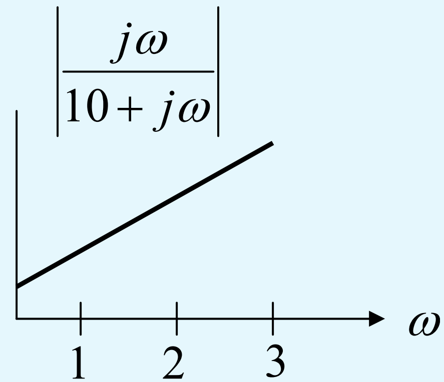
Demodulation of FM signals

– The differentiator can be approximated by any device whose magnitude transfer function is reasonably linear with the range of frequencies of interest.

Example



$$e_o(\omega) = \frac{j\omega L}{R + j\omega L} e_i(\omega)$$



(Note: $F\left\{\frac{df(t)}{dt}\right\} = j\omega F(\omega)$)